BPM Signal Processing: A Cartoon View

Rob Kutschke

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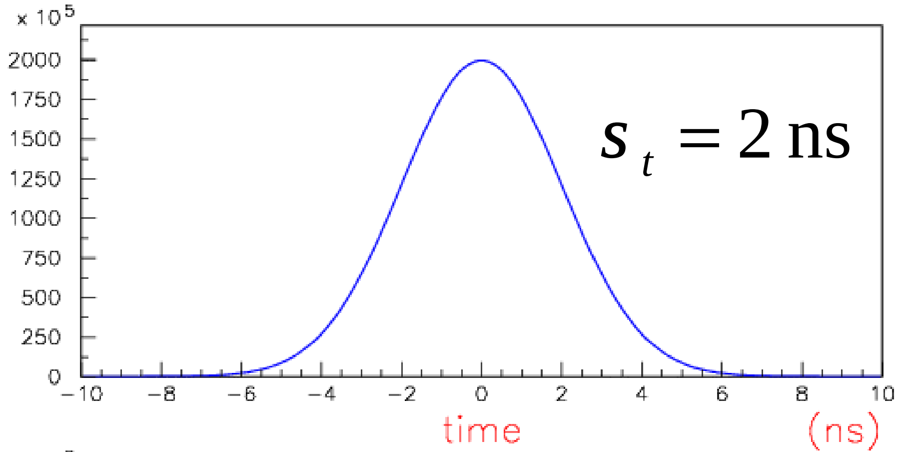
Outline

- Reminder about notation.
- Reminders about Fourier Transforms (FT).
- Bunch shape.
- Response of resonant filter to a single pulse.
- Response of resonant filter to multiple pulses, batch mode and bunch mode.
- A proposal about how to analyze the output of the resonant filter.

Reminder about Notation

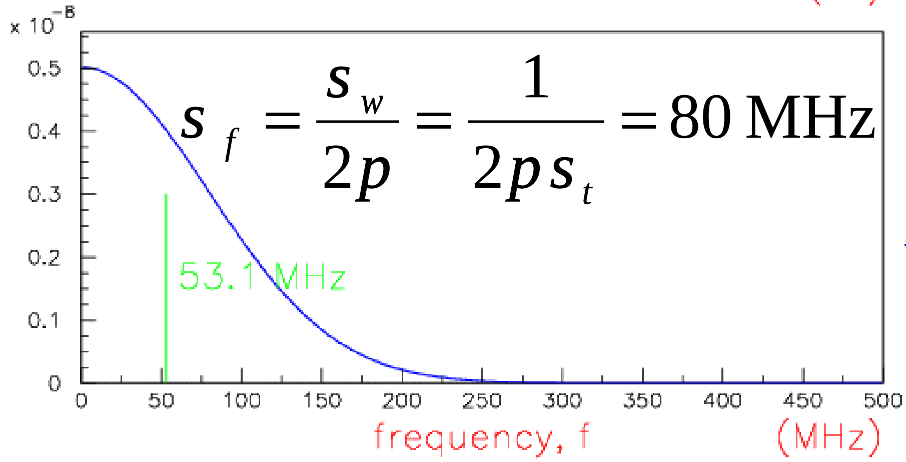
- $F(t) = \sin(2\pi f_0 t) = \sin(\omega_0 t)$
- Frequency: f_0
- Angular frequency: $\omega_0 = 2\pi f_0$
- Period: $T_0 = 1/f_0 = 2\pi/\omega_0$
- When someone says "frequency" they usually mean f_0 but sometimes they mean ω_0 !

FT of a Gaussian is Gaussian



$$s_t = 2\ \text{ns}$$

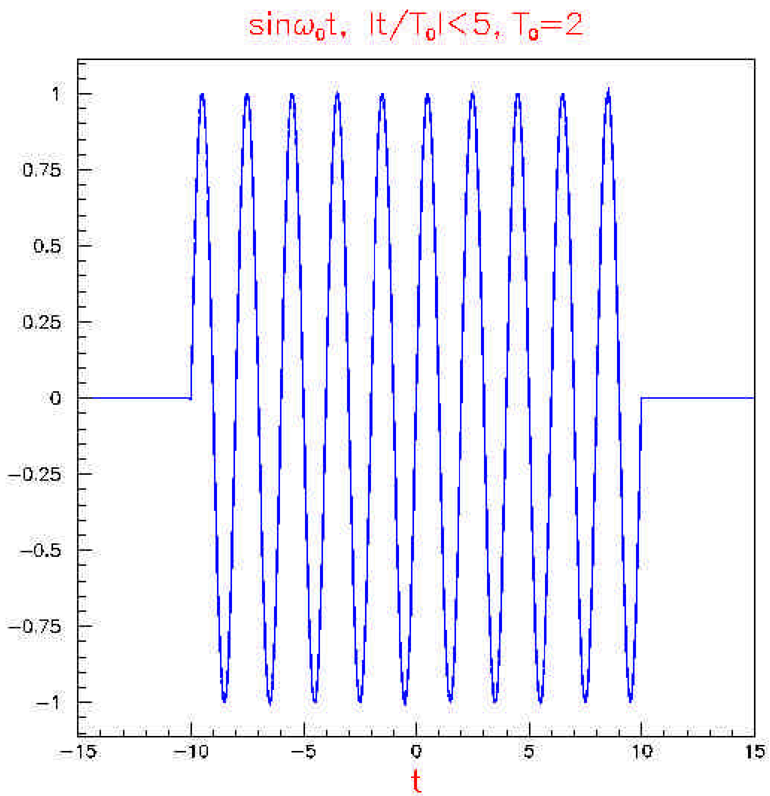
Bunch shape

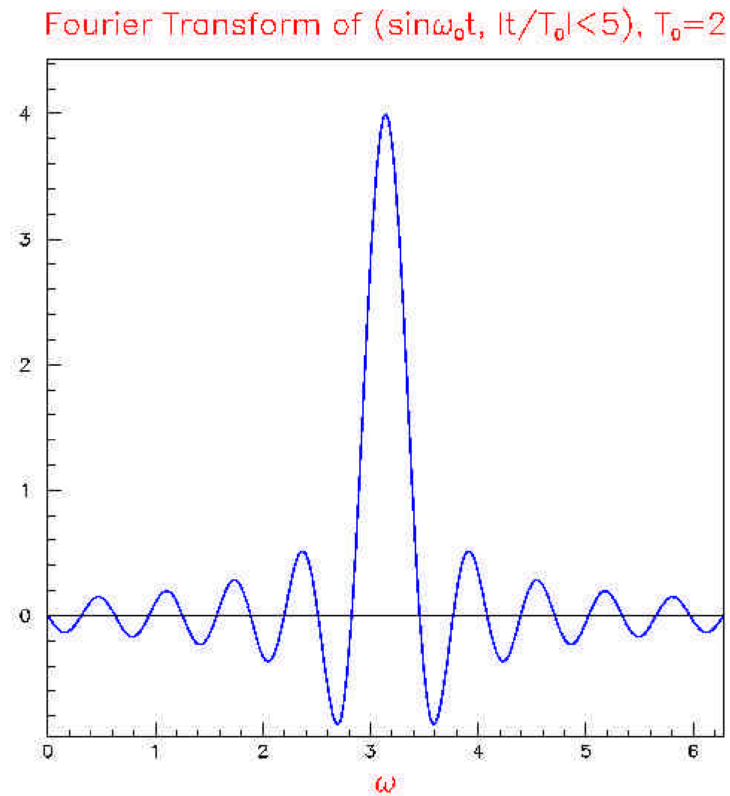


$$s_f = \frac{s_w}{2p} = \frac{1}{2ps_t} = 80\ \text{MHz}$$

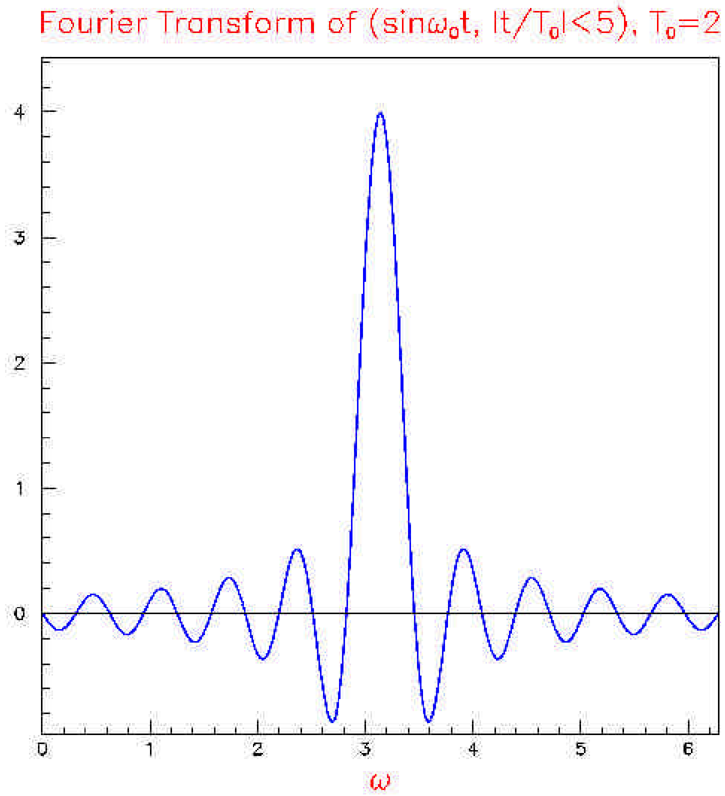
Power spectrum of a single bunch passing a fixed point.

FT of Finite Wave Train





FT of Finite Wave Train



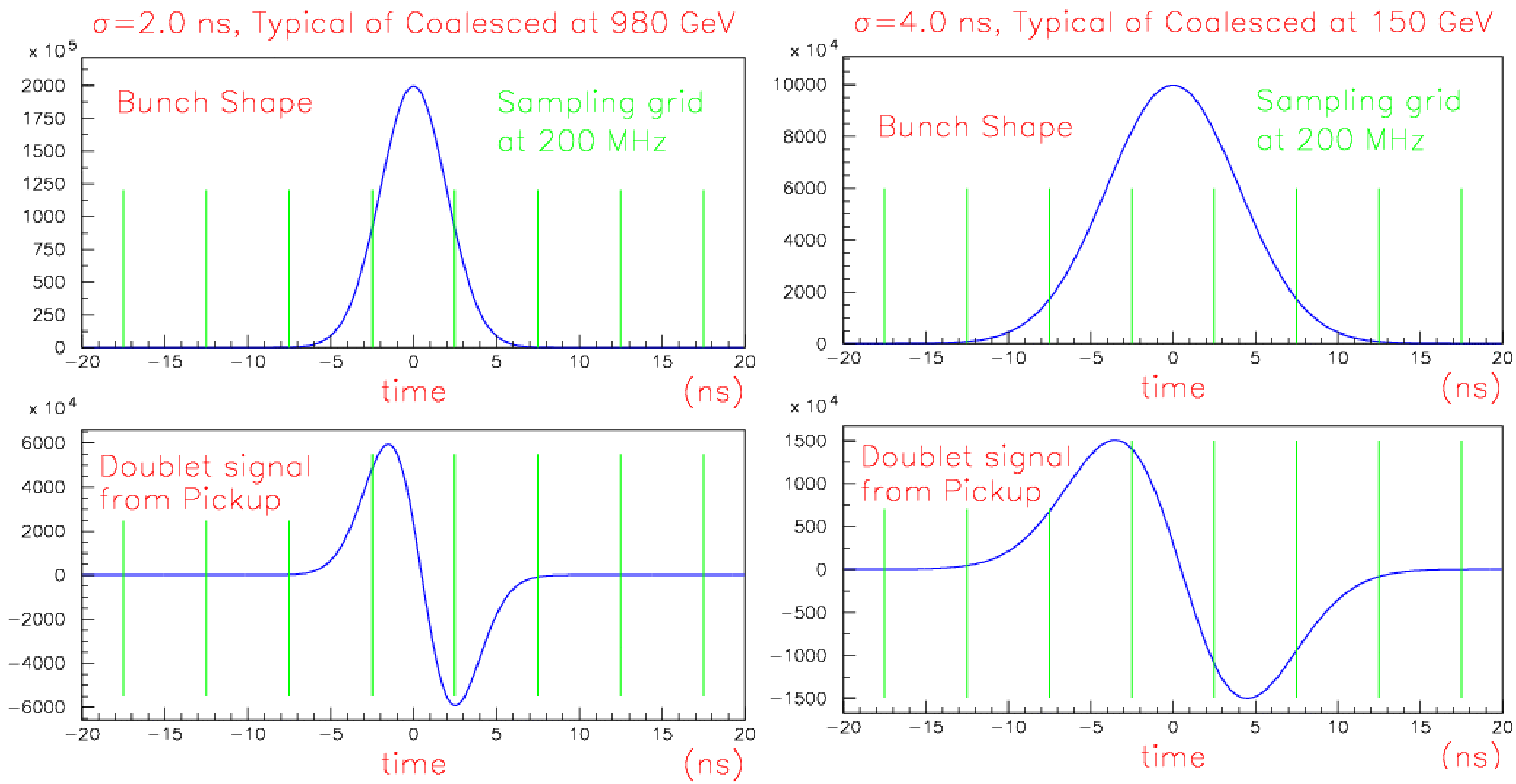
- Define N = number of oscillations in the time domain (= 10 here).
- First zero near max is at: $(\delta\omega/\omega_0) = \pm 1/N$ ($\omega_0 = \pi$, in this example)
- Equivalent statement: zero occurs when the number of oscillations in the interval differs by ±1.

Bunch Shapes

- Shape of a single bunch is nominally a Gaussian.
  - Never really is.
- Nominal scaling of bunch length is 1/sqrt(E).
  - In real life bunches do get shorter with energy but slower than this.
- Coalesced bunches:
  - $\sigma \approx 4$ ns at 150 GeV (injection energy)
  - $\sigma \approx 2$ ns at 980 GeV
- Uncoalesced bunches
  - About 1/9 as many particles/bunch.
  - $\sigma \approx 2$ ns at 150 GeV (injection energy)
  - $\sigma \approx 1$ ns at 980 GeV

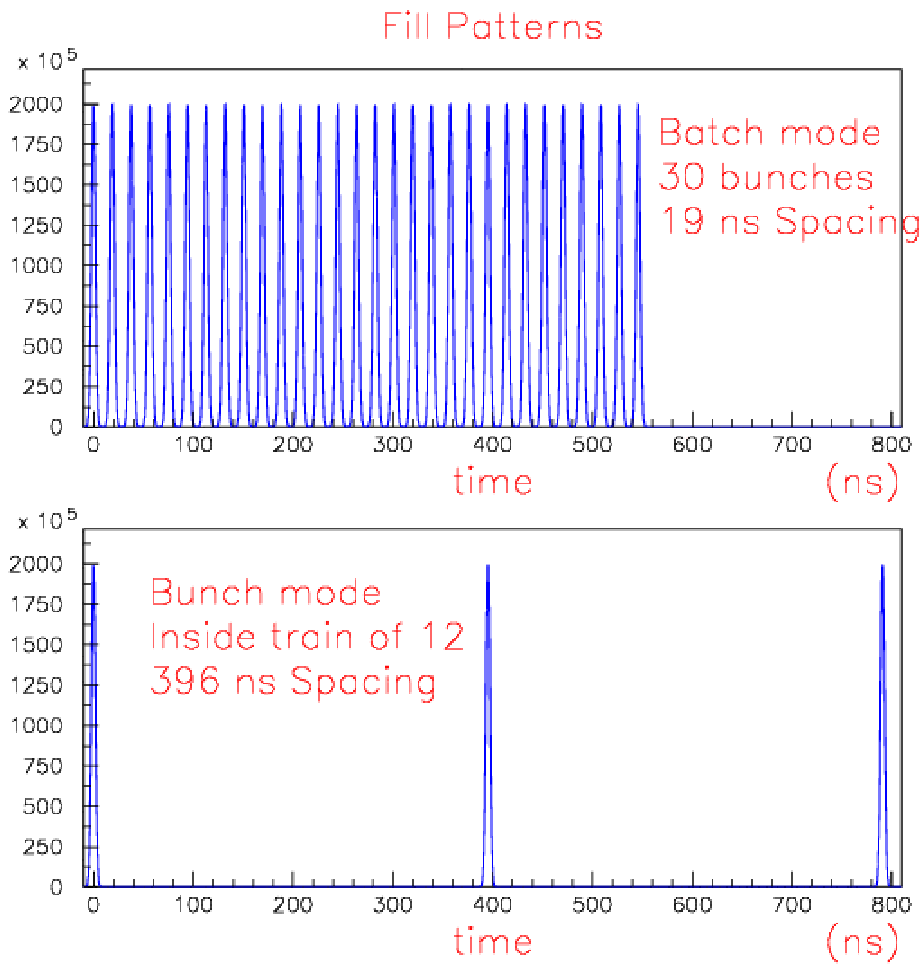
Coalesced vs Uncoalesced

- Coalescing: take n bunches, separated by 19 ns and put them on top of each other.
  - Typically n=9.
  - Nominal behaviour:
    - Number of particles in the coalesced bunch is n times the number in each uncoalesced bunch.
    - Time width increases as sqrt(n)
      - This ideal case is usually not achieved and bunch width is a little wider.



- Is ≈200 MHz is fast enough to digitize pulse with no shaping?
- A and B plates must be digitized at same time: time difference implies a position error.

Three Fill Patterns of Interest



- Third pattern is just a single bunch in the machine.
- In batch mode there is only ever one batch in the machine. Number of bunches in a batch may change.
- Present bunch mode is 3 trains of 12 bunches. The new BPM should allow other choices.

Required Measurements

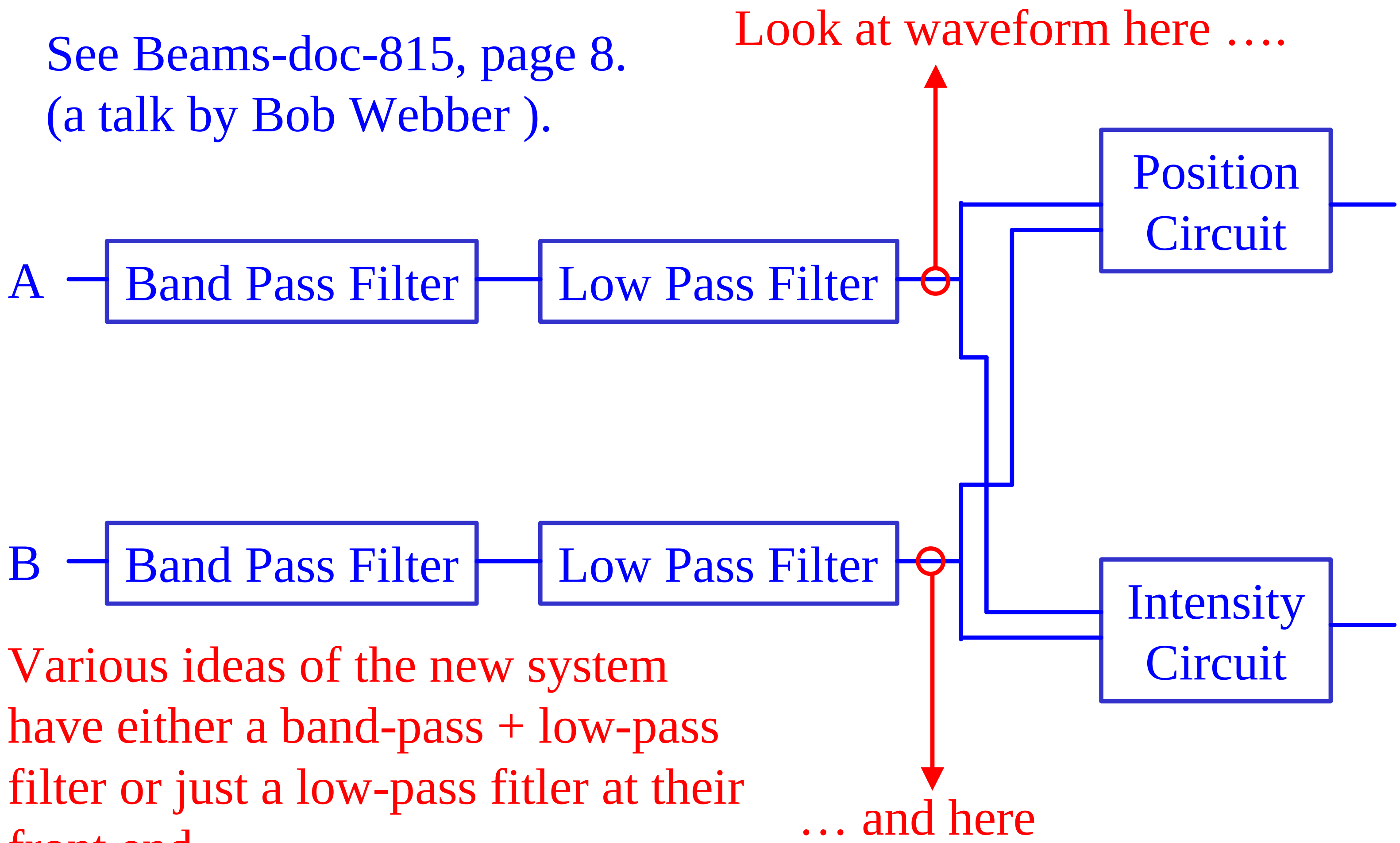
- First turn:
  - Only needed for a single bunch in the machine.
  - Report turn by turn positions for first turn.
- Turn by turn:
  - Report turn by turn position.
  - Only needed for a single bunch or single batch in the machine.
- Closed orbit:
  - Means to average over many turns to average out the betatron oscillations.
  - Needed for all fill patterns.

Cartoon of Our Options

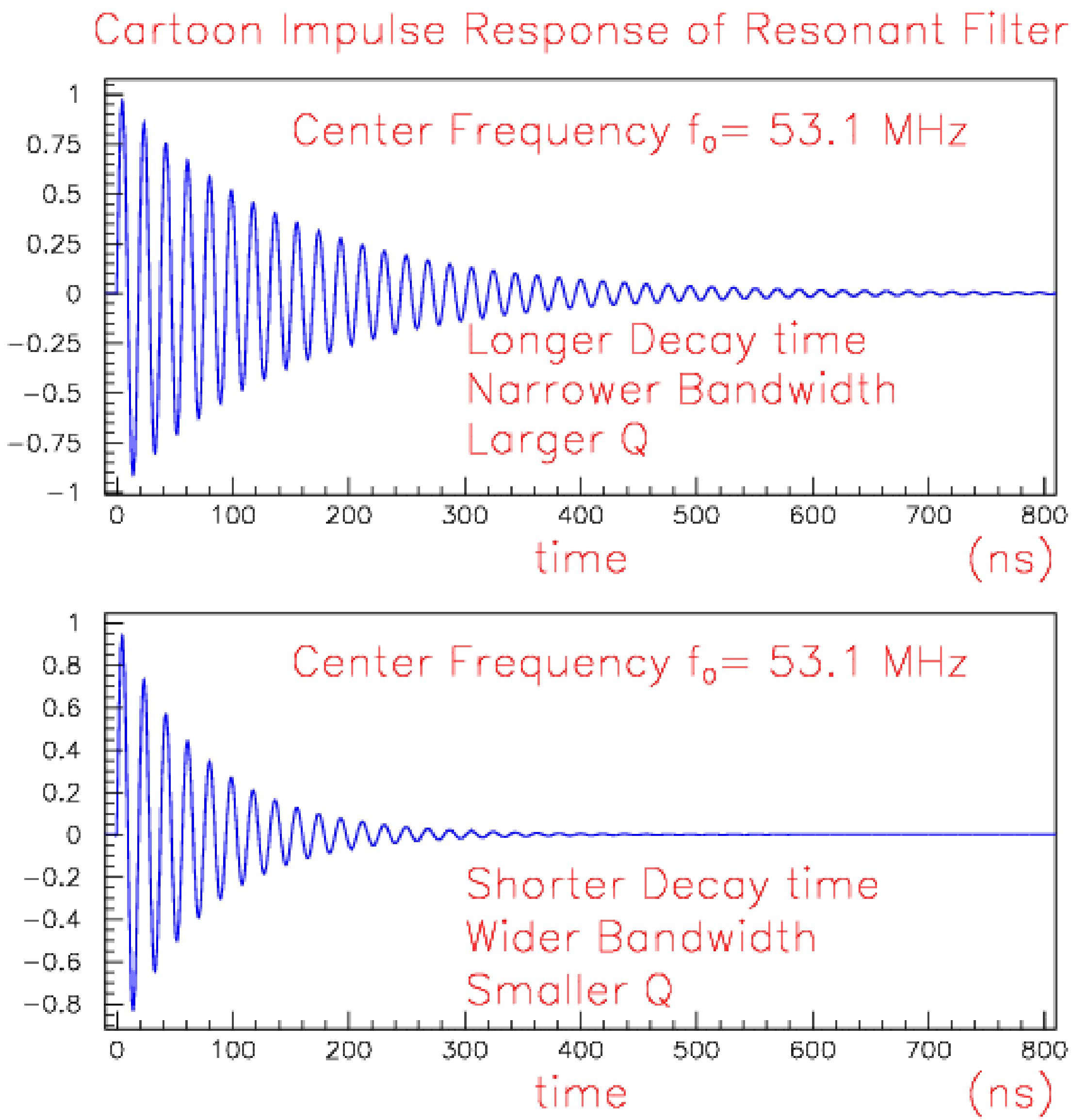
- Superfast digi, no shaping (2 GHz or more).
  - Very expensive.
- Shape the pulse (Digitize once or many).
  - Need to work hard at timing? Especially on first turn?
  - Tradeoff: long pulse good for precision and accuracy but bad for separation of protons and anti-protons.
    - How to deal with batch mode when bunches are 19 ns apart?
- Ring resonant filter and transfer position info to the frequency domain.
  - Coherent addition of bunch signals is natural.
  - Give up single bunch resolution.

Existing RF Module

See Beams-doc-815, page 8.
(a talk by Bob Webber).

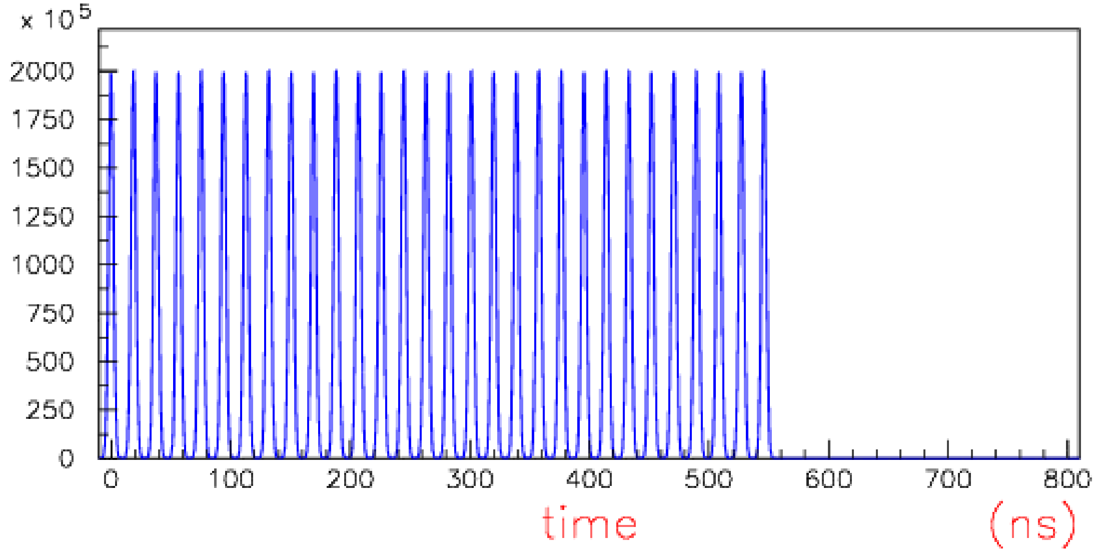


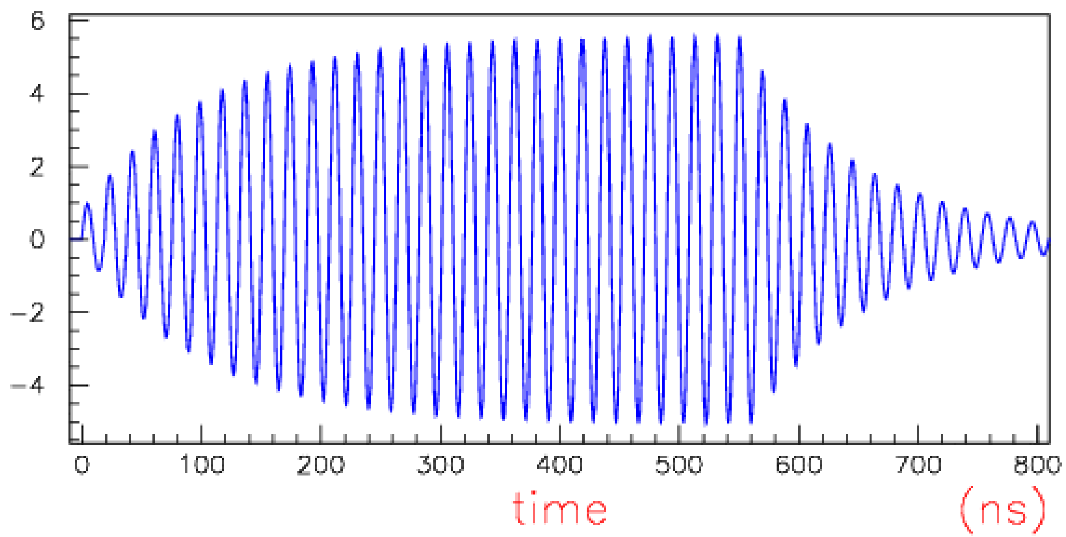
Various ideas of the new system have either a band-pass + low-pass filter or just a low-pass fitler at their front end.



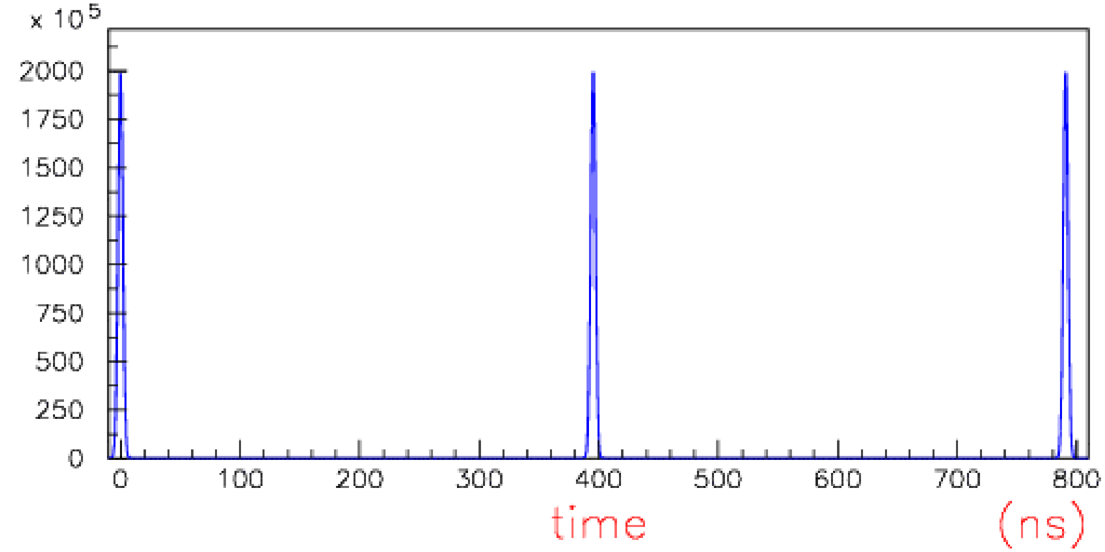
Cartoon shows the 53.1 MHz component only. The true signal will be complicated by other frequencies inside the band-pass.

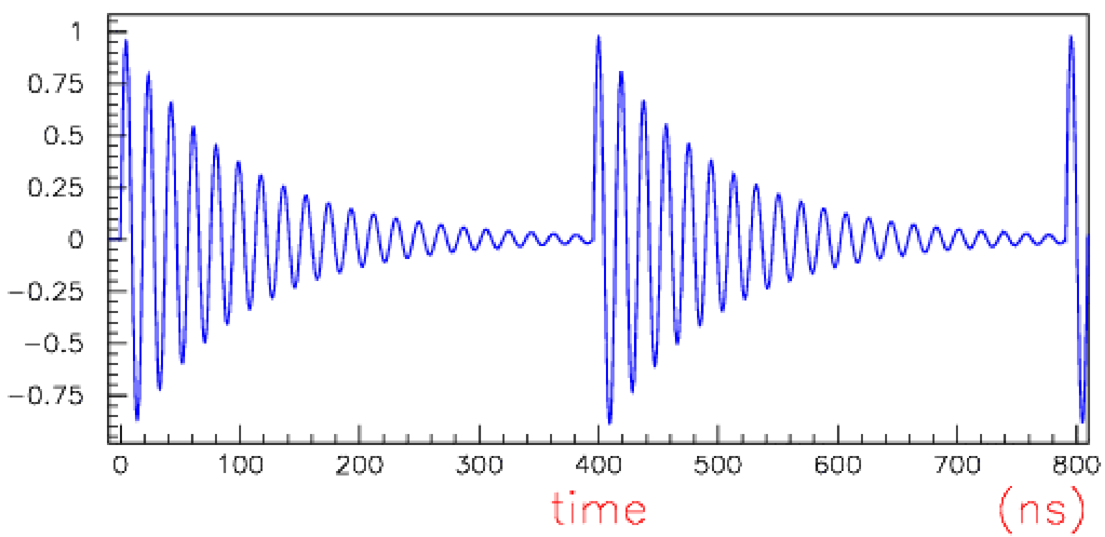
Why 53.1 MHz? How many rings do we want/need? Answers later.





- Bunch pattern in batch mode.
- Bunches are uncoalesced.
- Cartoon response of the resonant filter, to the above batch of bunches.
- Existing BPM electronics designed to use this signal.

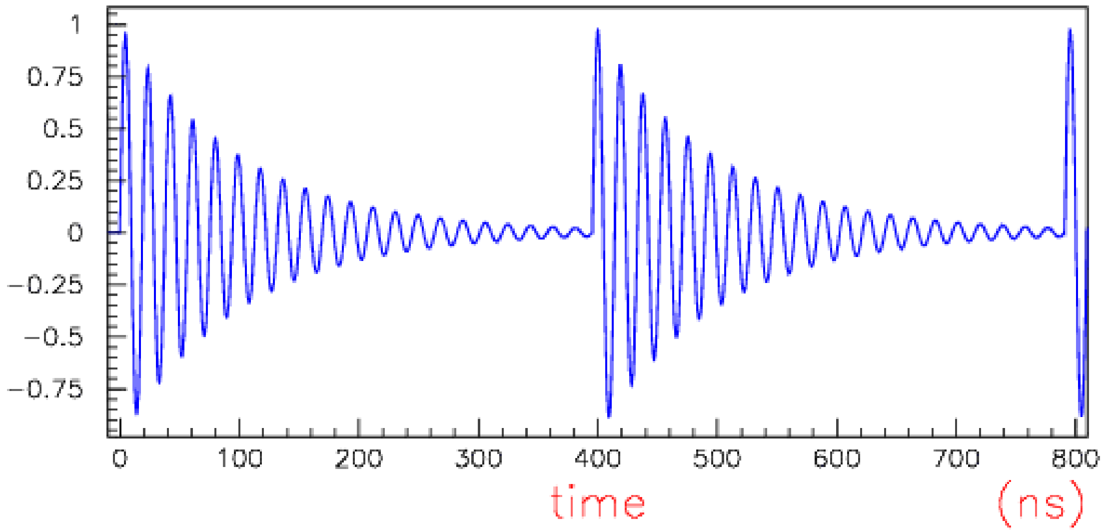




- Bunch pattern in “bunch mode”, 396 ns between bunches in a train.
- Bunches are coalesced.
- Cartoon response of the resonant filter, to the above batch of bunches.

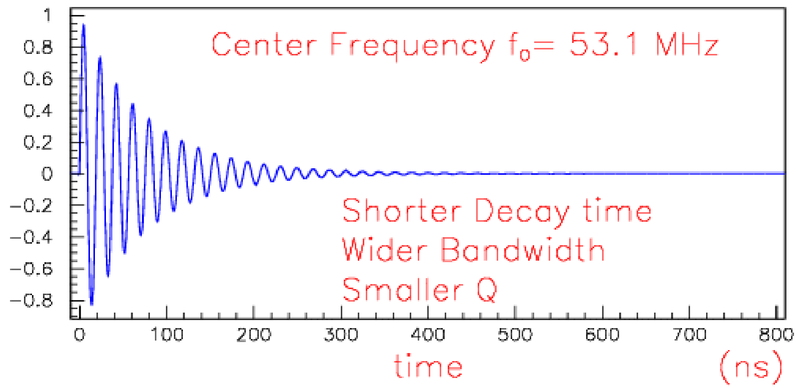
Comment on 53.1 MHz

- The 53.1 MHz component of the filter is kicked in phase by each passing bunch.
  - Also true for harmonics of 53.1 MHz.
- All other frequencies will be excited out of phase by each bunch.
  - Wave form at these frequencies is more complex and either we lose information or we need fancier signal processing.
- Following slides will talk about extracting the information at 53.1 MHz.



- How to process the above waveform to get a position?
  - A and B signals differ in amplitude.
- For now, consider just protons or anti-protons in the machine.
- Also assume no significant reflections.

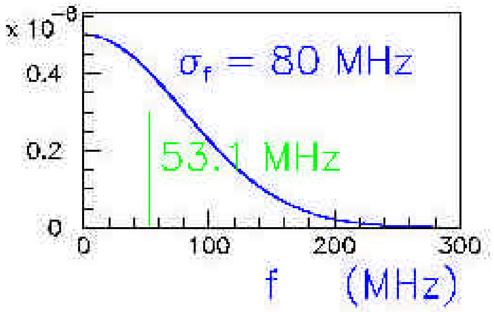
Time to Frequency Domain



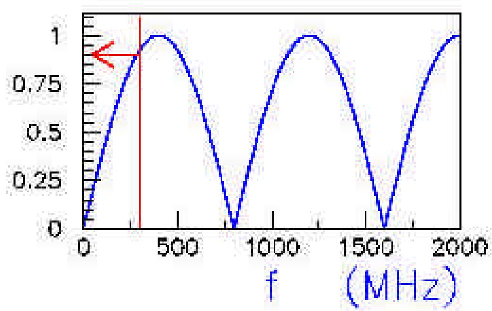
Details of this shape will not be important.

- Single pulse, coming out of the front end filters. It contains:
  - The time structure of the beam.
  - Modified by the response of the pickup, cables and front end filters.
  - Call this shape: $f(t)$
- Can also say the same things in frequency domain.
  - Call that shape: $\tilde{f}(w)$

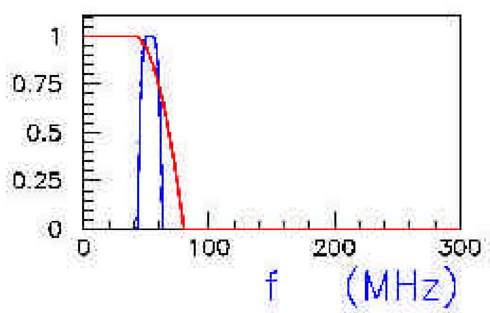
Cartoons of Frequency Response



Frequency content of a single gaussian bunch.



Frequency response of pickup (Beams-doc-766, page 10).
Line matches scale in top plot.



Frequency response of:
- band pass filter
- low pass filter

$\tilde{f}(w)$ Is the product of these shapes.

Comments on Previous Slide

- Really should also draw the phase information for these plots.
- The full function will be slowly varying in some neighbourhood about 53.1 MHz.
  - Not sure what the phase does in this region or if it is important?

Single Bunch, Multiple Turns

- Time dependence of signal after filter of a single bunch which never changes shape and which follows a closed orbit past the same point N times:

$$A(t) = \sum_{n=0}^{N-1} A_0 f(t - t_n)$$

- If orbit is exactly periodic:

$$t_n = nT_0 = \frac{n}{f_0}$$

Same Thing, Frequency Domain

- Fourier Transform of a single pulse:

$$\tilde{f}(w) = \int_{-\infty}^{\infty} f(t)\exp(iwt)dt$$

- FT of the full waveform:

$$\tilde{A}(w) = \int_{-\infty}^{\infty} A(t)\exp(iwt)dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \left[A_0 f(t - t_n)\right]\exp(iwt)dt$$

$$\int_{-\infty}^{\infty}\sum_{n=0}^{N-1}[A_0 f(t-t_n)]\exp(iwt)dt$$

Repeat last equation on previous page.

$$= A_0\sum_{n=0}^{N-1}\int_{-\infty}^{\infty} f(t-t_n)\exp(iwt)dt$$

Reorder sum and integral.

$$= A_0\sum_{n=0}^{N-1}\exp(iwt_n)\int_{-\infty}^{\infty} f(t-t_n)\exp(iw(t-t_n))dt$$

Insert: $\exp(iw(t_n - t_n))$

$$= A_0\sum_{n=0}^{N-1}\exp(iwt_n)\boxed{\int_{-\infty}^{\infty} f(u)\exp(iwu)du}$$

Change variables, u = t - t_n

$$= A_0\boxed{\tilde{f}(w)}\boxed{\left[\sum_{n=0}^{N-1}\exp(iwt_n)\right]}$$

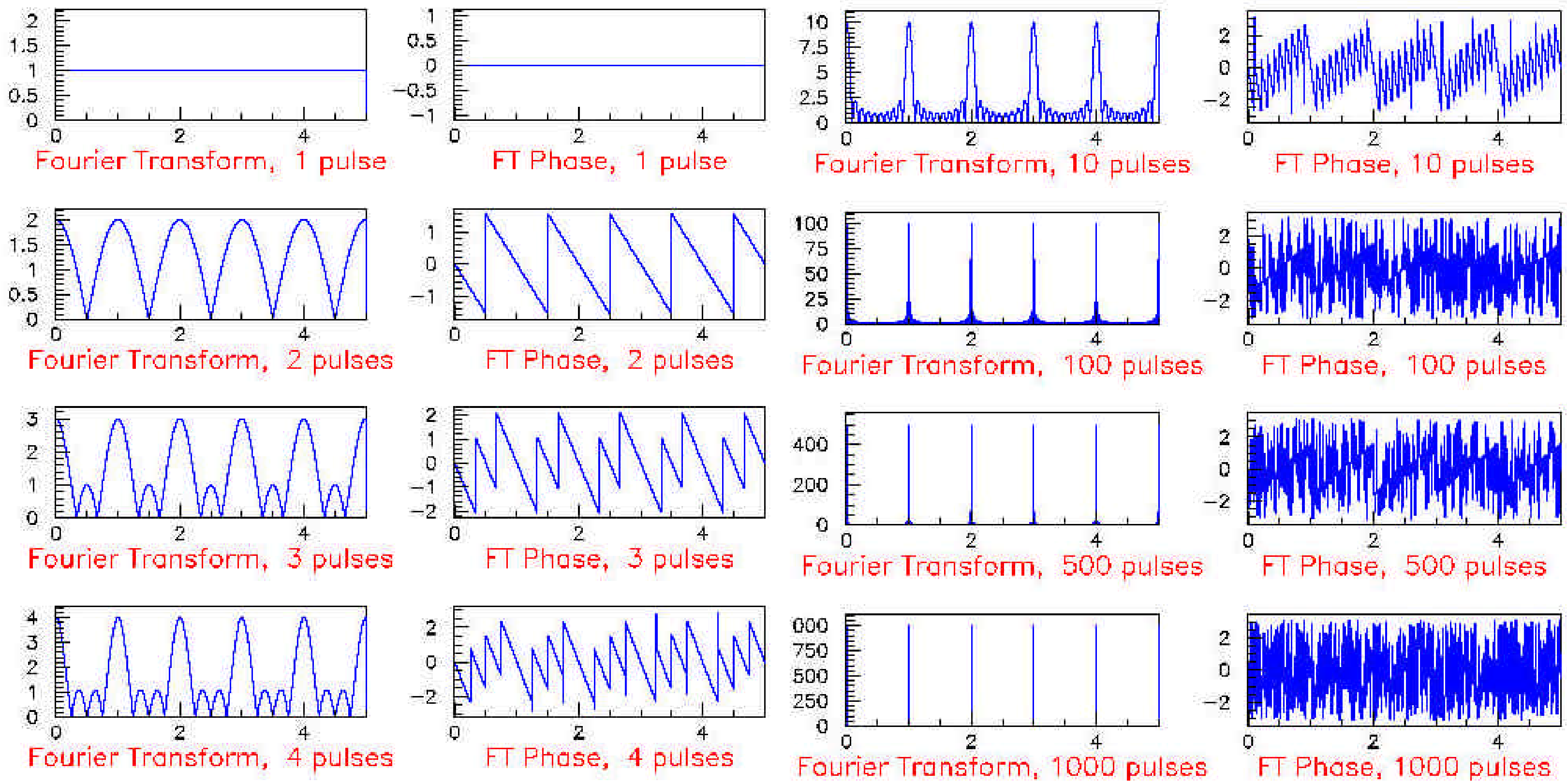
FT of the single pulse shape comes out of the sum!

Explore this shape for $t_n = nT_0$. For large N it will be a comb.

Toy Model of: $\sum_{n=0}^{N-1} \exp\left(i w n T_0\right)$

- The model with f_0 = 1 Hz
  - No signal for a long time.
  - N copies of one pulse, each separated by T_0
  - No signal for a long time.
- No edge effects of the FT window being too close to the first or last pulse.
- Try different values of N.

Fourier Transforms of $\sum_{n=0}^{N-1} \exp(iwnT_0)$ for different N



These functions multiply the FT of a single pulse.

Add Betatron Oscillations

$$A(t) = \sum_{n=0}^{N-1} A_n f(t - t_n)$$

$$t_n = nT_0$$

$$A_n = A_0 + \boxed{a_0 \sin(w_b t_n + d_b)}$$

- A(t) = signal on the A plate.
- B(t) = Signal on B plate will have opposite phase.

- Sinusoid dependence of A_n may be an approximation?
- This has the approximation that betatron phase is constant over the time of the bunch (2-4 ns).

Do the math with $A_0 \rightarrow A_n$

$$\int_{-\infty}^{\infty} \sum_{n=0}^{N-1} [A_n f(t-t_n)] \exp(iwt) dt$$

Repeat last equation on previous page.

$$= \sum_{n=0}^{N-1} A_n \int_{-\infty}^{\infty} f(t-t_n) \exp(iwt) dt$$

Reorder sum and integral.

$$= \sum_{n=0}^{N-1} A_n \exp(iwt_n) \int_{-\infty}^{\infty} f(t-t_n) \exp(iw(t-t_n)) dt$$

Insert: $\exp(iw(t_n - t_n))$

$$= \sum_{n=0}^{N-1} A_n \exp(iwt_n) \boxed{\int_{-\infty}^{\infty} f(u) \exp(iwu) du}$$

Change variables, u = t - t_n

$$= \tilde{f}(w) \boxed{\left[\sum_{n=0}^{N-1} A_n \exp(iwt_n) \right]}$$

FT of the single pulse shape cancels out.

Now look at this shape for betatron oscillations.

Betatron Oscillations

$$\sum_{n=0}^{N-1} A_n \exp(iwt_n)$$

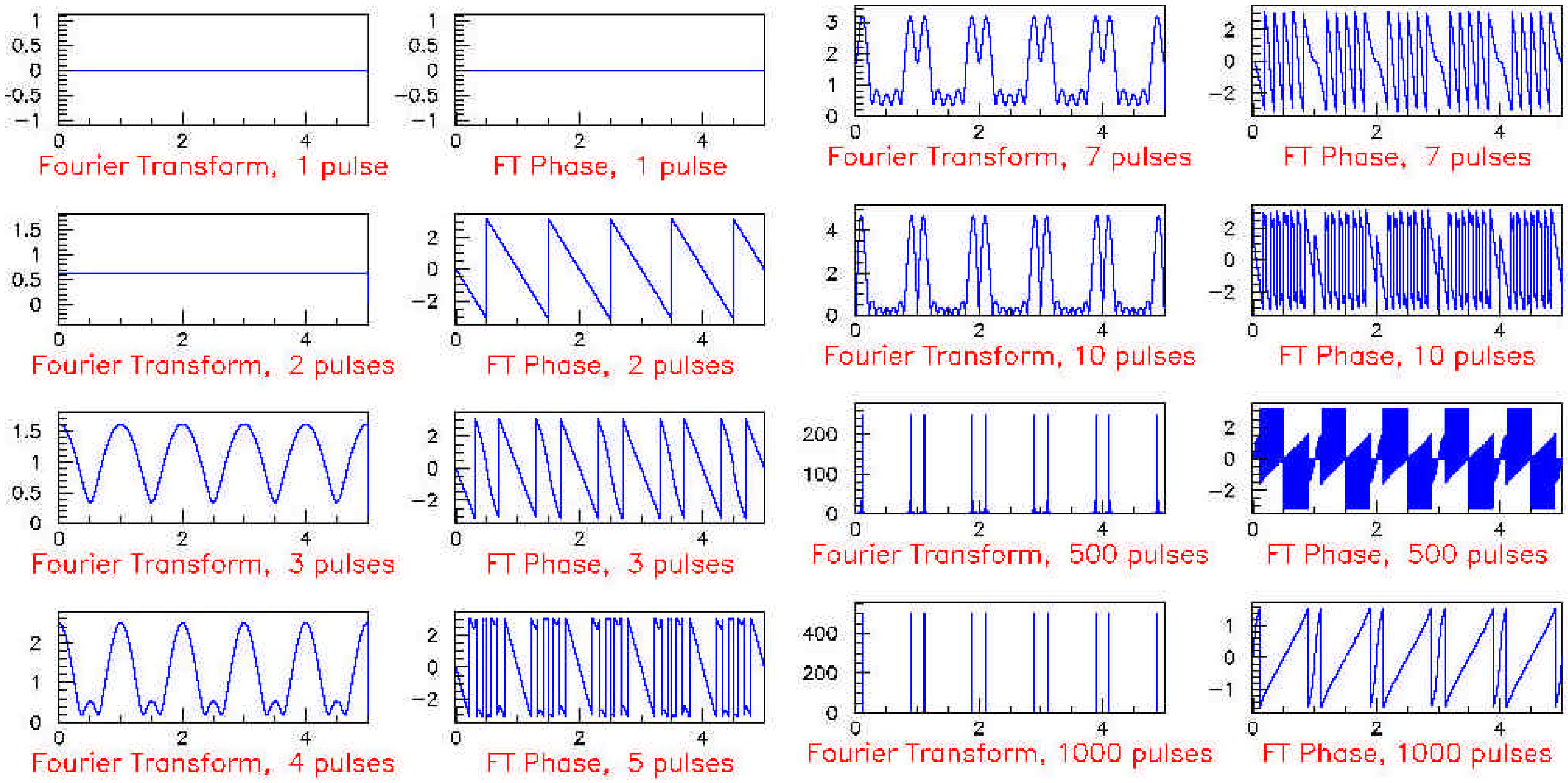
$$= \sum_{n=0}^{N-1} \left(A_0 + a_0 \sin(w_b nT_0 + d_b)\right) \exp(iwnT_0)$$

We have seen the first term already. Look at the second term.

$$= a_0 \sum_{n=0}^{N-1} \sin(w_b nT_0 + d_b) \exp(iwnT_0)$$

Extend the previous toy model. f_0=1 Hz, f_β=10 Hz, $\delta_\beta = 0$

Fourier Transforms of $\sum_{n=0}^{N-1} \sin(w_b n T_0) \exp(i w n T_0)$ for different N



Sidebands appear at $\omega = \omega_0 \pm \omega_\beta$.

Add Synchrotron Oscillations

$$A(t) = \sum_{n=0}^{N-1} A_n f(t - t_n)$$

$$t_n = nT_0 \boxed{+ t_0 \sin(w_s nT_0 + d_s)}$$

$$A_n = A_0 + a_0 \sin(w_b t_n + d_b)$$

- Approximation that the synchroton phase is constant across the bunch, (2-4 ns).
- Won't go any farther with this for now. Also will create a sideband structure.

Two identical bunches, separated in time by δ

$$A(t) = \sum_{n=0}^{N-1} A_0 f(t - t_n) + \sum_{m=0}^{M-1} A_0 f(t - t_m)$$

$$t_n = nT_0 \qquad t_m = mT_0 + d$$

$$FT(w) = A_0 \tilde{f}(w) \left[\sum_{n=0}^{N-1} \exp(iwt_n) + \sum_{m=0}^{M-1} \exp(iwt_m) \right]$$

$$= A_0 \tilde{f}(w) [1 + \exp(iw\, d)] \left[\sum_{n=0}^{N-1} \exp(iwnT_0) \right]$$

Still under construction.

Two Different Bunches

$$A(t) = \sum_{n=0}^{N-1} A_n f(t - t_n) + \sum_{m=0}^{M-1} A_m g(t - t_m)$$

$$FT(w) = \tilde{f}(w) \sum_{n=0}^{N-1} A_n \exp(iwt_n) + \tilde{g}(w) \sum_{m=0}^{M-1} B_m \exp(iwt_m)$$

First Turn Mode

- Needs to work only for one bunch in the machine.
- For each of A and B signals:
  - Digitize at ≈ 200 MHz.
  - Compute FFT of digitized time series.
    - Integration time can be as long as a full turn if the filter output after one turn is still above the noise and least count.
    - Long integration time = narrow band.
  - Output of FFT is the power in a frequency band centered at 53.1 MHz. This is A or B.
- Position = (A-B)/(A+B)

Question

- I think that a long integration time in the FFT implies more precise values of A and B, which implies more precise position? Is this right?
- At the same time, it changes the meaning of A and B since the bandwidth decreases.

Turn by Turn Mode

- Only required to work for single bunch or single batch in the machine.
- Requirements say that it is OK to average over bunches:
  - How many?
  - Report this average on each turn.

Turn by Turn mode

- Works as for first turn but the integration time window of the FFT may be changed.
  - Might integrate over signal from one bunch, from several, … up to all 36 bunches.
    - Integration over several bunches produces some sort of average position.
  - Question: if there are no bunch to bunch diseases, does accuracy improve if integration time is longer?

Closed Orbit

- Must work for all fill patterns.
- As before but integration time of FFT is many turns.
  - This averages out the betatron oscillations.
- Questions:
  - If we integrate a long time the bandwidth narrows - can we lose important information located in the sidebands? Maybe A becomes a histogram covering a range of frequencies, not just a single number. Then do peak finding on it?

What About Anti-Protons?

- For now, consider proton signal corrupting the anti-proton measurement.
- When the anti-proton intensity problems are solved, we will also need to worry about the antiprotons corrupting the proton measurement.

What About Anti-Protons?

- Now we get a waveform from each end of each plate. Processes each the same way as described before.
- In the limit of perfect directionality, we directly measure: A_P, A_{Pbar}
  - Real numbers, the output of each FFT.
- For imperfect directionality, the observed numbers are contaminated with each other.

Imperfect Directionality

- Define two feedthrough coefficients: ε_1, ε_2.
- Also need to know relative phase, δ, between proton and pbar waveforms.
  - Depends on state of cogging.
- These differ from one BPM to the next.
- The instrument measures:

$$\alpha_P = \left| A_P + \varepsilon_1 e^{i\delta} A_{Pbar} \right|$$

$$\alpha_{Pbar} = \left| A_{Pbar} + \varepsilon_2 e^{-i\delta} A_P \right|$$

Imperfect Directionality

$$\alpha_P = \left| A_P + \varepsilon_1 e^{i\delta} A_{Pbar} \right|$$

$$\alpha_{Pbar} = \left| A_{Pbar} + \varepsilon_2 e^{-i\delta} A_P \right|$$

- Measure α_P and α_{Pbar}, to get two equations and two unknowns.
  - Solve for A_P, A_{Pbar}.
- Need to know ε_1, ε_2, and δ for each BPM and each cogging.
- Similarly to extract B_P, B_{Pbar}.

Questions

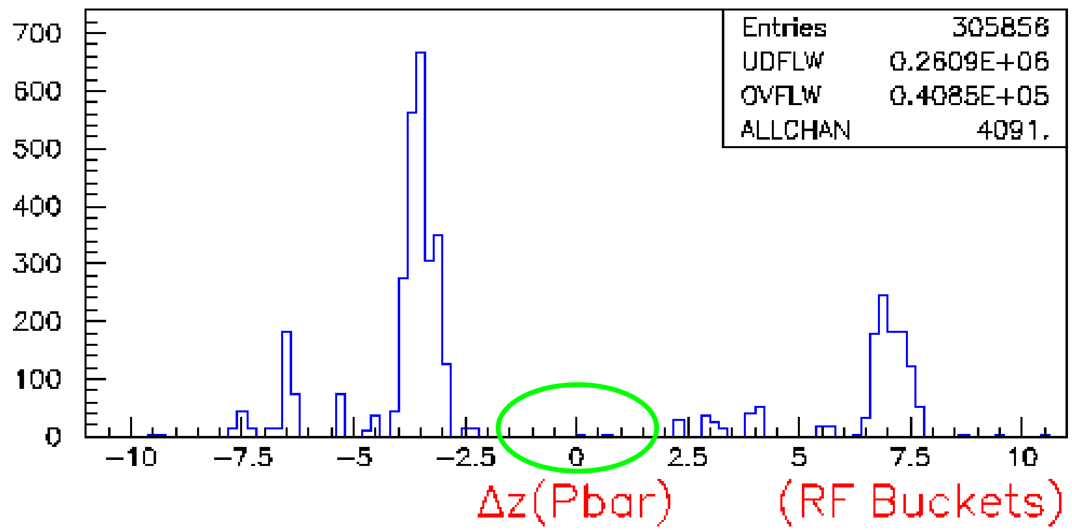
- The 2-species response also depends on the relative phases of the betatron and synchrotron oscillations of the p and pbar? Is this a signifcant effect? Is it stable? Is it repeatable from store to store?
- Closed orbit is probably immune from this regardless of its size, but not turn by turn?

Second Comment on 53.1 MHz

- Could this work with other frequencies ?
- Sure, but.
  - Only at 53.1 MHz and its harmonics is the interference between bunches in phase for all fill patterns.
  - At other frequencies, some fill patterns will excite destructive interference, which reduces the sensitivity of the measurement.
- Could do 53.1 MHz for batch mode and 53.1/21 MHz for bunch?

Back to Short Pulse

- The alternative to the ringing filter is to stretch the pulse out a little and then sample it (either single measurement or 200 MHz sample).
- Tightest time window is set by proton antiproton separation.



- For each BPM, compute arrival time of proton bunch.
- Compute position of all antiproton bunches at that time.
- Plot position difference of P and Pbar.
- Above figure:
  - For "Collision Point Cogging", P1=A1 at F0.
  - Not sure of sign.
- Only 6 instances with separation < 2.35 buckets (44.5 ns).

P/Pbar Overlaps for Collision Point Cogging

BPM	Proton	AntiProton
HPA0U	1	13
HPA0U	25	25
HPB0U	1	25
VPA0U	1	13
VPA0U	25	25
VPB0U	1	25

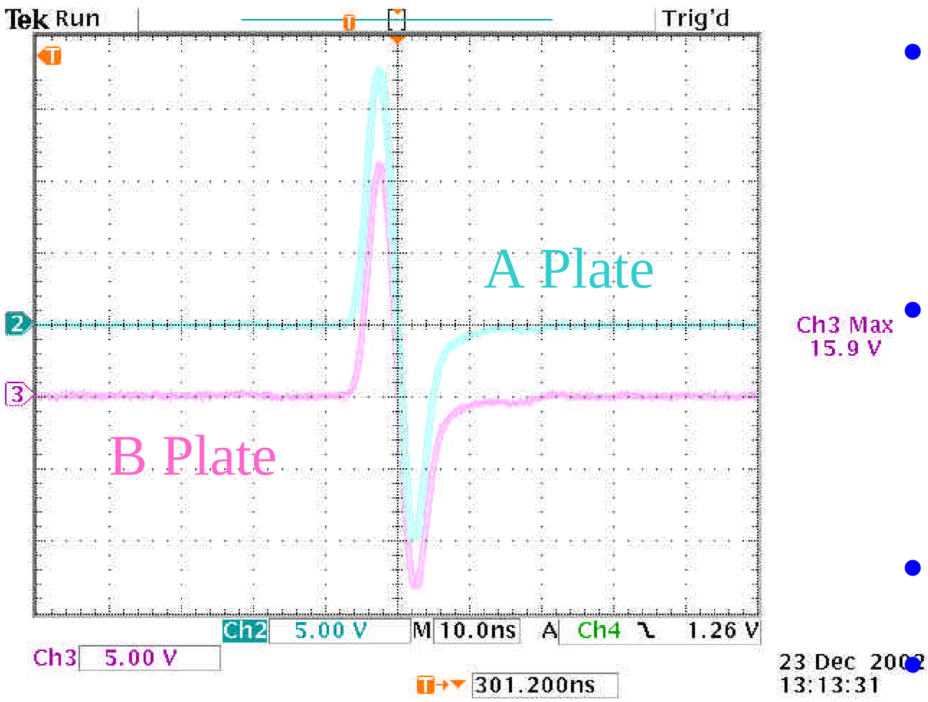
Protons and Anti-Protons

- Define shapes of after pickup, cables and filters
  - f(t): proton signal from the proton end.
  - g(t): anti-proton signal on the proton end.

$$A(t) = \sum_{n=0}^{N-1} P_n f(t - t_n) + \sum_{m=0}^{M-1} A_m g(t - t_m)$$

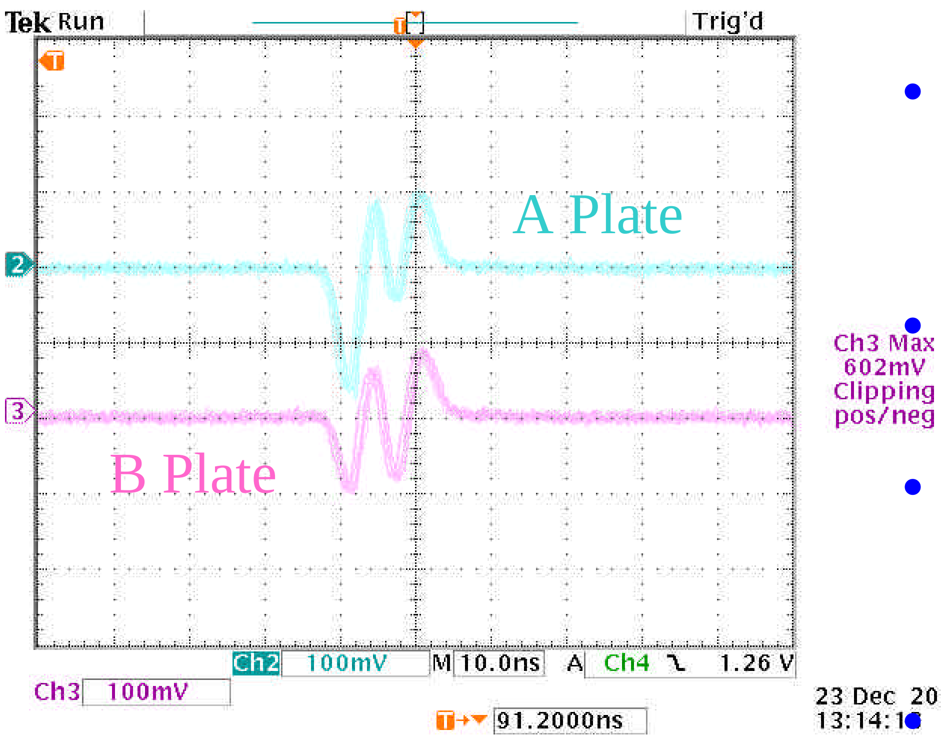
$$\tilde{A}(w) = \tilde{f}(w)\left[\sum_{n=0}^{N-1} A_n \exp(iwt_n)\right] + \tilde{g}(w)\left[\sum_{m=0}^{M-1} A_m \exp(iwt_m)\right]$$

Backup Slides



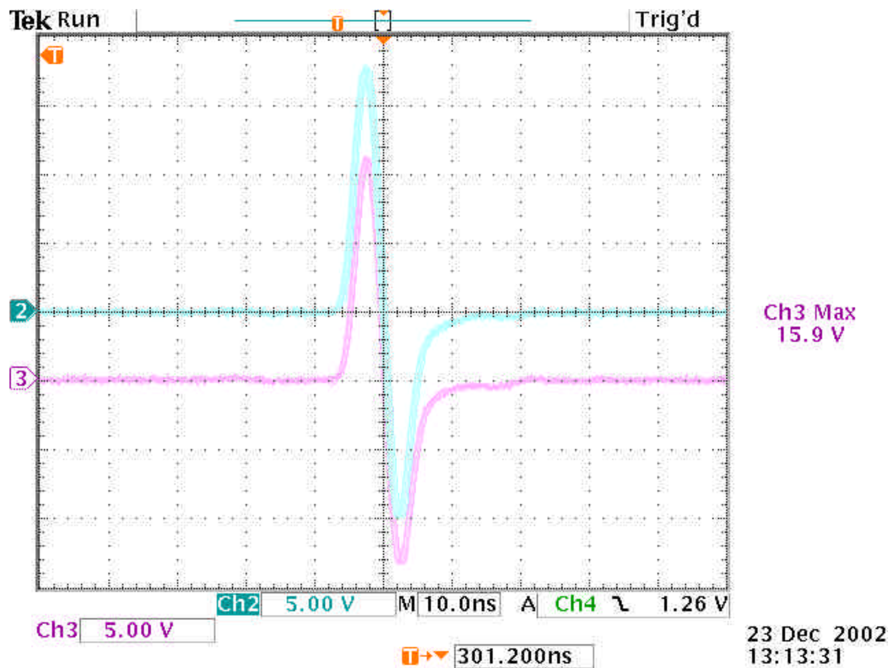
- Scope picture from Fritz D.
  - Uses HPA17
  - Signals on cables in house.
  - No filters or attenuators

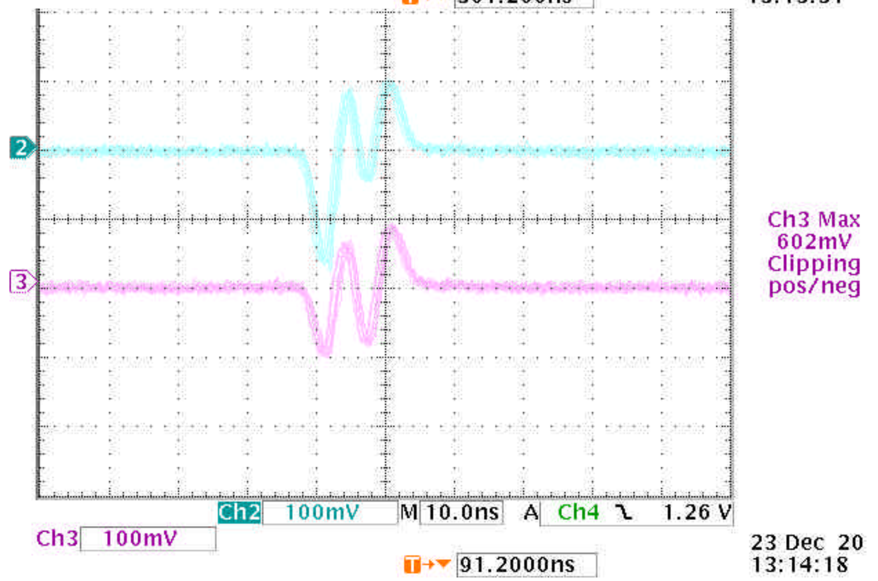
- Signal on proton end from proton bunch in a regular store.
- Tiggered either by "intensity signal" or by a TeV marker???
- Very little ringing.
- FWHM of positive piece is about 4 ns.
  - Gaussian: $\sigma \approx 1.7$ ns.



- Scope picture from Fritz D.
  - Uses HPA17
  - Direct from cables in house.
  - No filters or attenuators

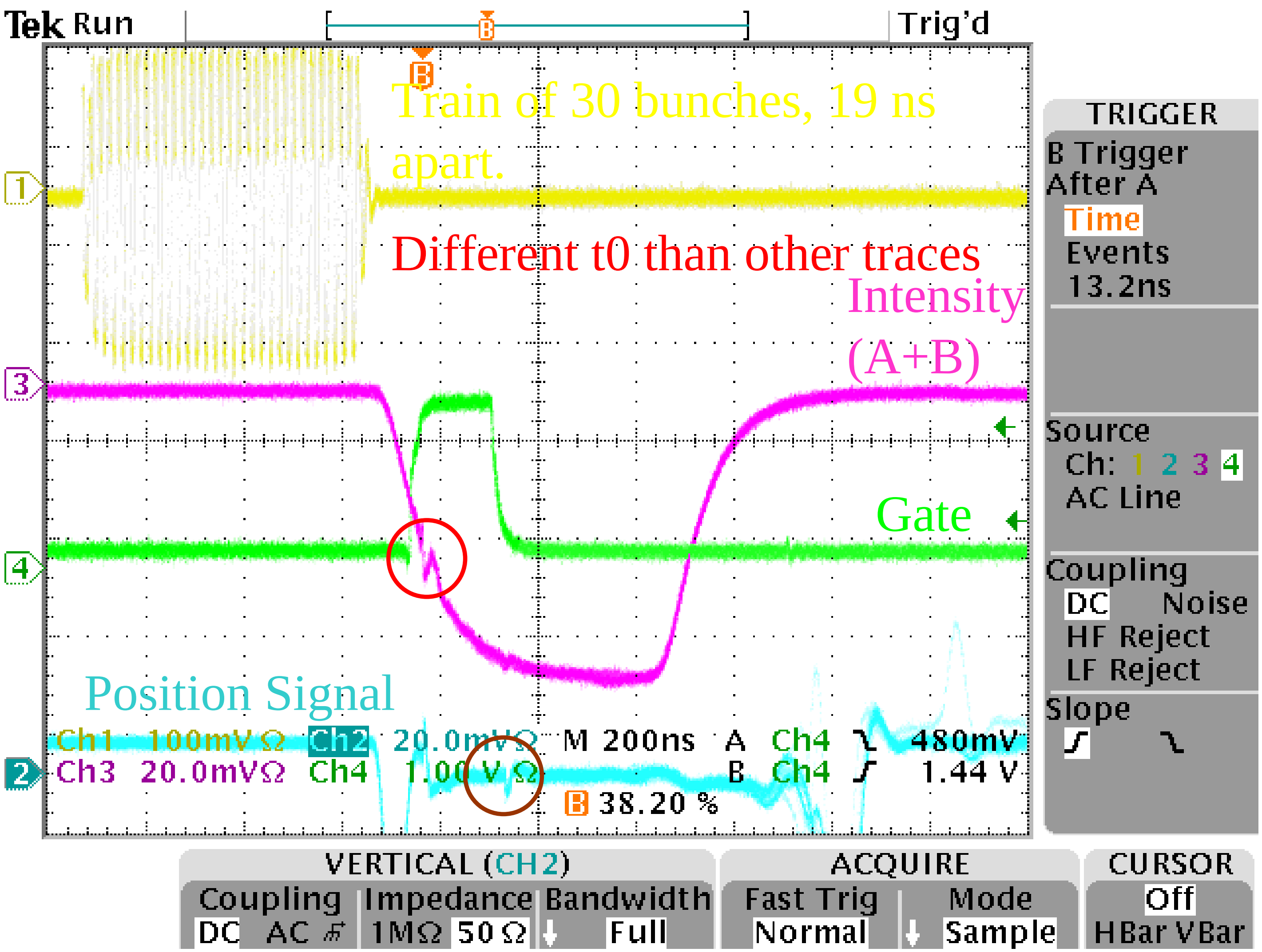
- Signal on anti-proton end induced by the proton bunch.
- Tiggered same way as previous slide.
- Unsure how to interpret timing wrt previous slide.
- As before:
  - Very little ringing.
  - FWHM of first negative piece ≈ 4 ns.


Tek Run
Trig'd
2
3
Ch3 Max
15.9 V
Ch2 5.00 V
M 10.0ns
A Ch4 1.26 V
Ch3 5.00 V
301.200ns
23 Dec 2002
13:13:31


2
3
Ch3 Max
602mV
Clipping
pos/neg
Ch2 100mV
M 10.0ns
A Ch4 1.26 V
Ch3 100mV
91.2000ns
23 Dec 20
13:14:18

Comments for Next Slide

- This is a scope picture showing the response of a BPM to a train of about 30 uncoalesced bunches.
- Top trace shows signal on cables
- Bottom 3 traces characterize BPM response.
  – These are all in time wrt each other
  – The top trace has an unknown time shift.



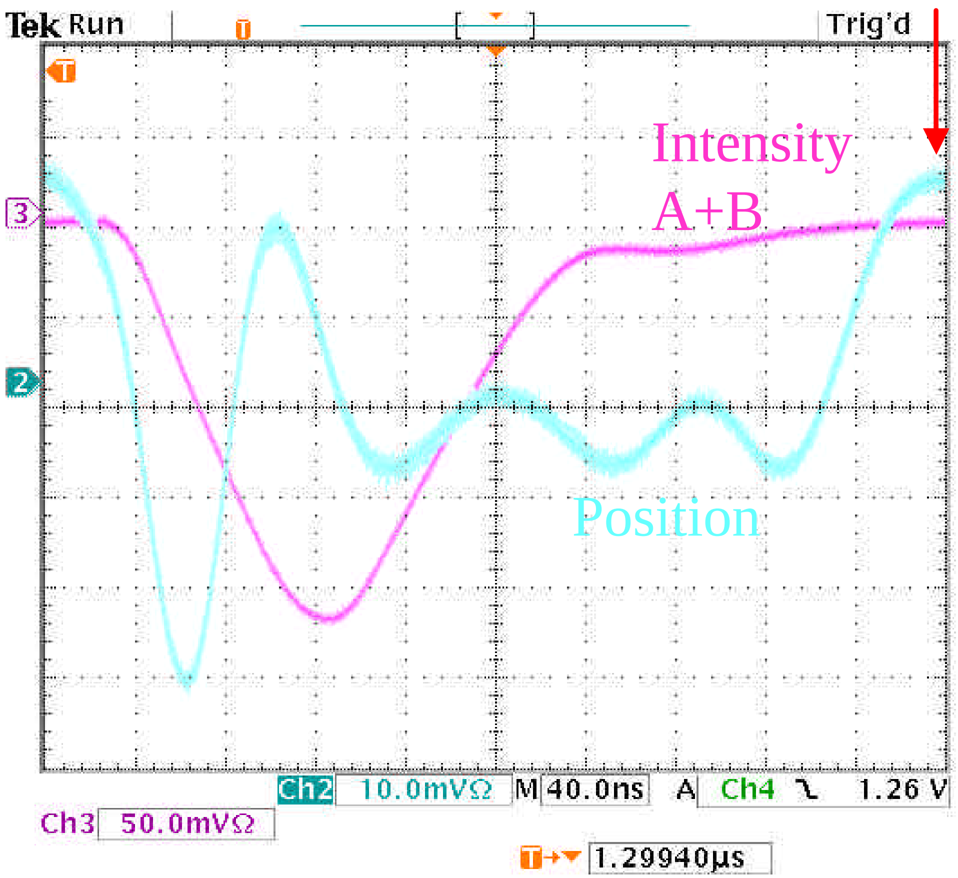
Tek Run
Trig'd
Train of 30 bunches, 19 ns apart.
Different t0 than other traces
Intensity (A+B)
Gate
Position Signal
Ch1 100mV Ω Ch2 20.0mV Ω M 200ns A Ch4 480mV
Ch3 20.0mV Ω Ch4 1.00 V Ω B Ch4 1.44 V
38.20 %
TRIGGER
B Trigger After A
Time
Events
13.2ns
Source
Ch: 1 2 3 4
AC Line
Coupling
DC Noise
HF Reject
LF Reject
Slope
VERTICAL (CH2)
Coupling
DC AC
Impedance
1MΩ 50 Ω
Bandwidth
Full
ACQUIRE
Fast Trig
Normal
Mode
Sample
CURSOR
Off
HBar VBar

Notes on Previous Slide

- The bottom 3 traces have the same timing.
  - The top trace should shift to the right by ??? ns.
- Rise time of intensity signal is about 200 ns.
- Gate internally generated by threshold on intensity signal. (circled glitch)
- Is gate width programmable????

- Position Signal:
  - Overshoot at small time.
  - Position signal stable after about 200 ns.
  - Sample and hold on falling edge of gate (circled glitch)
  - Why is overshoot at end not in the opposite direction from that at start?

BPM Response to a Single Coalesced Bunch



Next bunch comes at this time (396 ns).

Does this signal ring at longer times?

- Scope picture from Fritz D. (HPA17).
- Before removal of PSD boards. Now ringing in position signal is less.

Comments on Previous Slide

- Intensity Signal
  - Rises to peak in 80 ns
    - Much faster than batch mode.
    - Conclude: this signal is never fully developed.
- Is there ringing at larger times – if so, then one bunch talks to the next bunch.

- Position Signal:
  - Overshoot has same width as for batch mode.
  - Never really gets flat.
    - Integration time of sample+hold ≈ a few ns.
  - When does sample and hold take place? Earlier than for batch mode?

Constraints

- Two Possible answers:
  1. As short as possible to separate protons and anti-protons in the time domain.
  2. It does not matter so long as the center frequency is a harmonic of the shortest possible bunch spacing. It can even have significant power after the next bunch has already arrived.
     - This talk will concentrate on explaining how this works.